



LoopFest IX

Radiative Corrections for the LHC and Lepton Colliders

June 21-23, 2010

# Heavy Quarks & PDFs

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Conspirators:

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**K. Kovarik, T.P. Stavreva,**

**J. Owens, J. Morfin, C. Keppel, D. Soper ...**

Loopfest  
23 June 2010

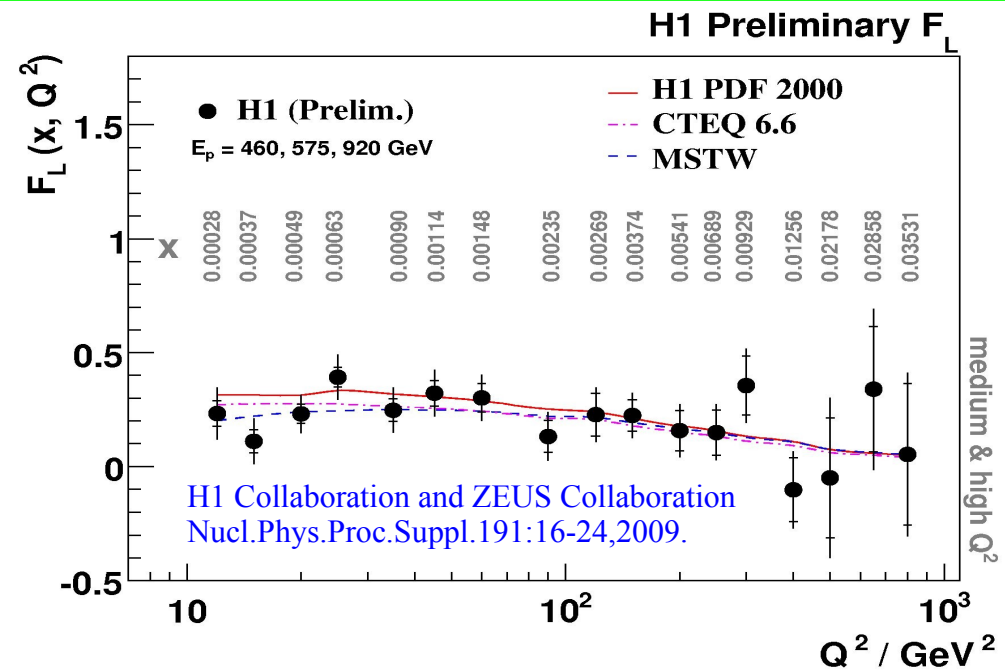
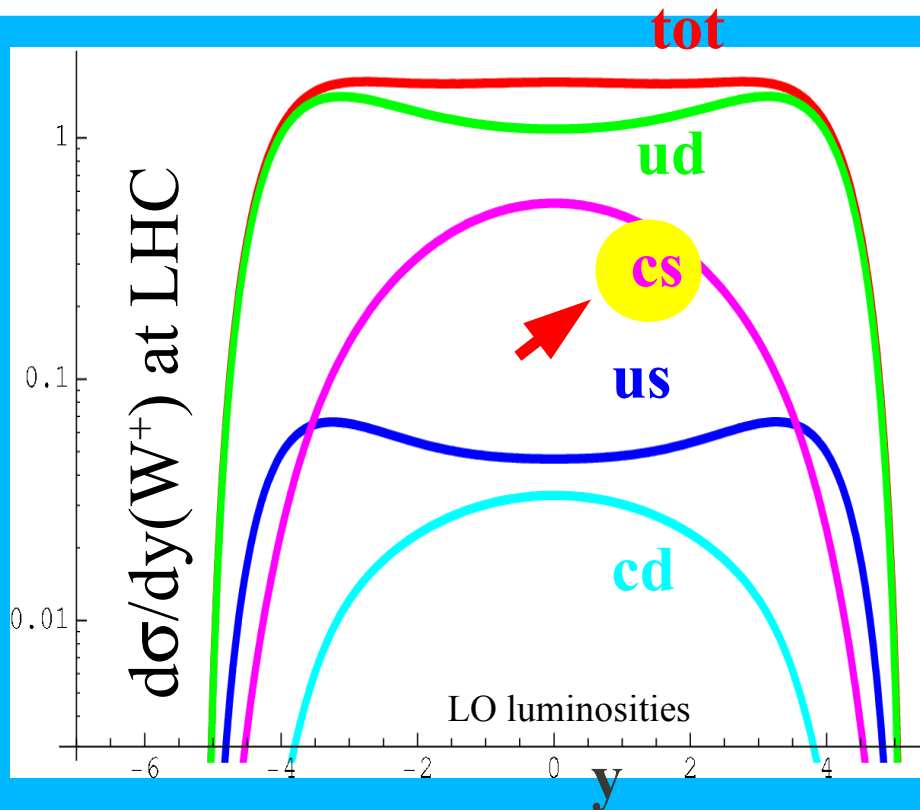
# Motivation: Heavy Quarks and the multi-scale problem

Two examples why heavy quarks are important

Mass terms can be leading effect

$$F_L \sim \frac{m^2}{Q^2} q(x) + \alpha_S \{ \dots \}$$

Masses are important



“Heavy Quarks” play a prominent role at the LHC

... heavy is a relative term  
Two scale problem

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# Good News

# GOOD NEWS: We have multiple schemes for heavy quark calculation

We have made progress in addressing how to compute heavy quarks. Recent efforts by many groups

## *The Cast:*

### ACOT & S-ACOT Codes

*Used in CTEQ4HQ, 5HQ, 6HQ*

Aivazis, Collins, Olness, Tung,  
Phys.Rev.D50:3102-3118,1994.

### S-ACOT

*CTEQ 6.5 & 6.6*

Tung, Lai, Belyaev, Pumplin, Stump, Yuan,  
JHEP 0702:053,2007.  
Nadolsky, Tung, Phys.Rev.D79:113014,2009.

### Thorne-Roberts (TR')

*MSTW Fits*

Thorne, Phys.Rev.D73:054019,2006.

### FONLL:

Used in NNPDF Fits

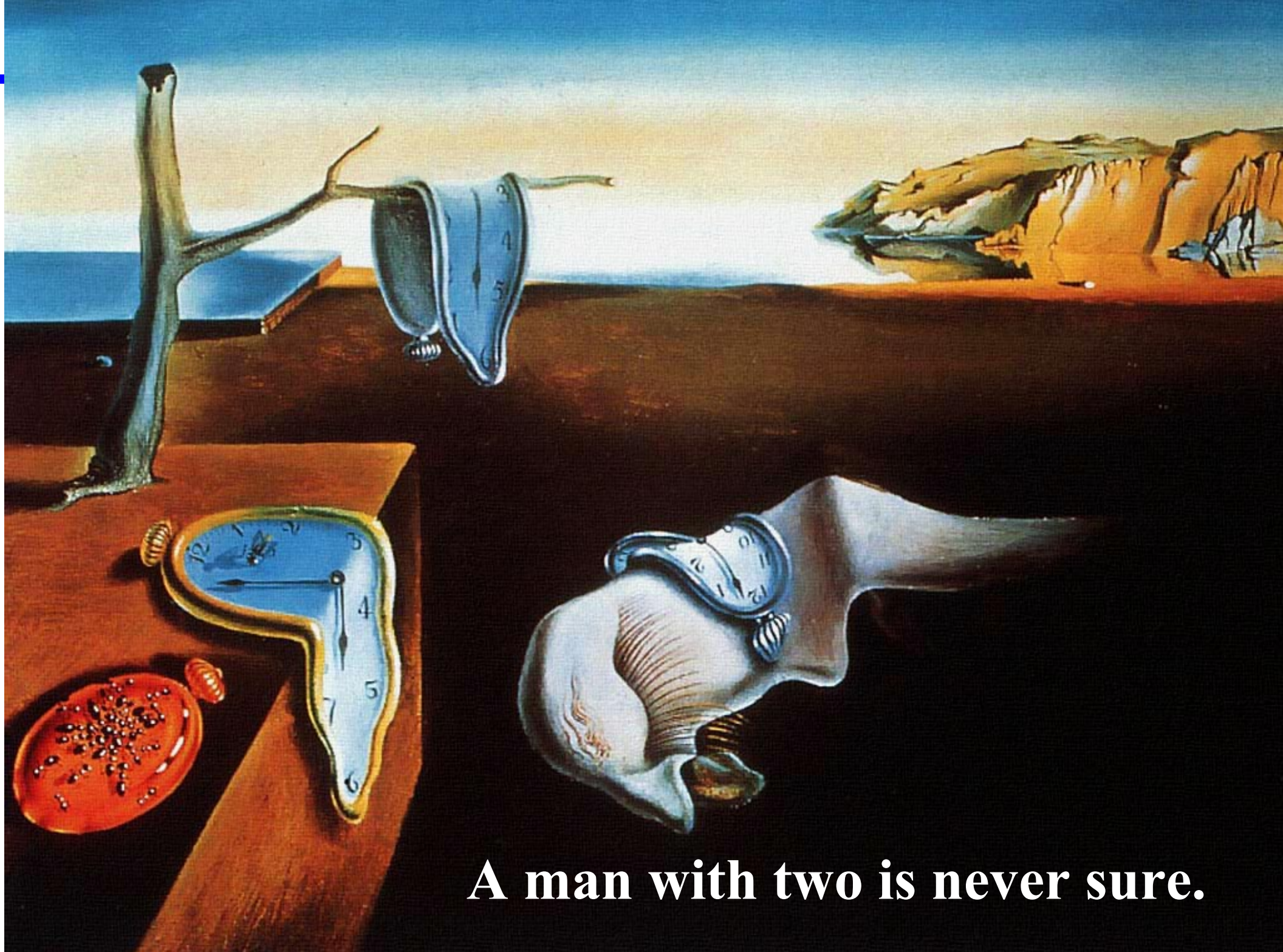
Forte, Laenen, Nason, Rojo,  
Nucl.Phys.B834:116-162,2010.

## **BUT: We have multiple schemes for heavy quark calculation**

A man with one watch  
knows what time it is ...







**A man with two is never sure.**



# 2009 Les Houches Comparative Studies

The SM and NLO Multileg Working Group: Summary report.

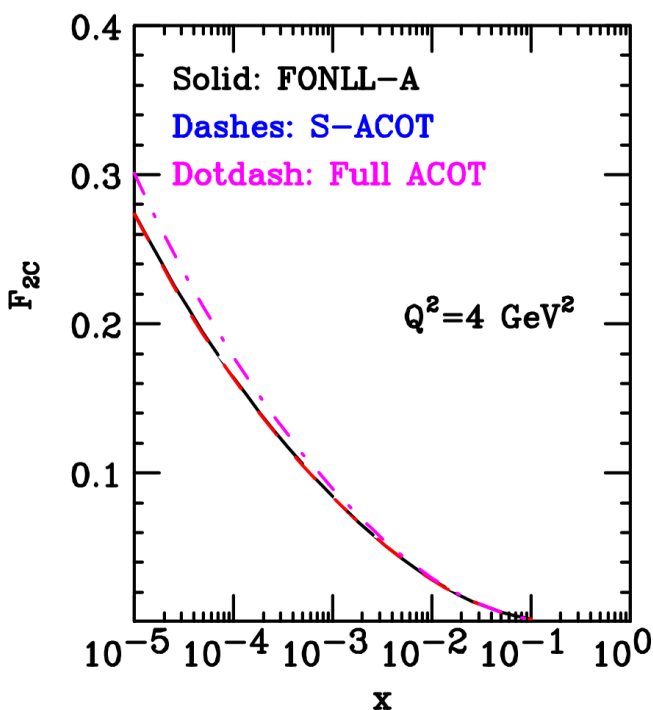
*e-Print: arXiv:1003.1241 [hep-ph]*



**Physics at TeV Colliders**  
Les Houches 8-26 June 2009

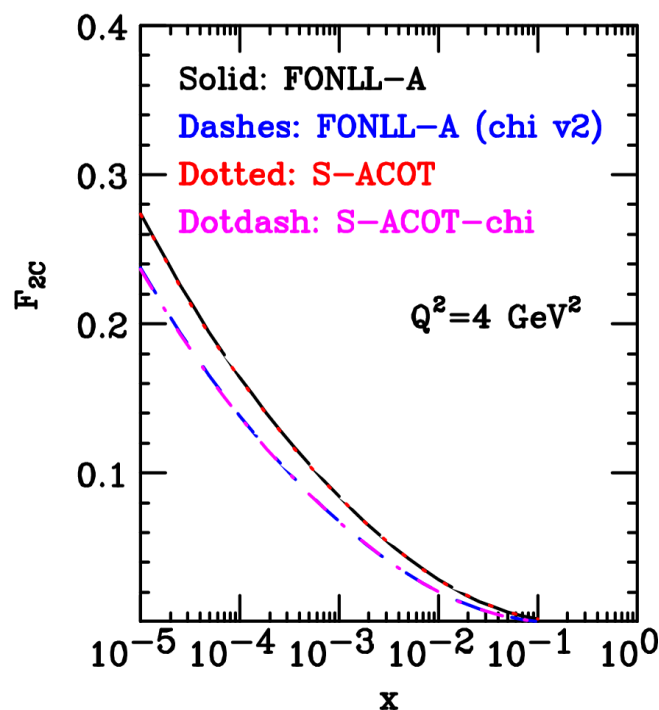


# Les Houches Comparative Study



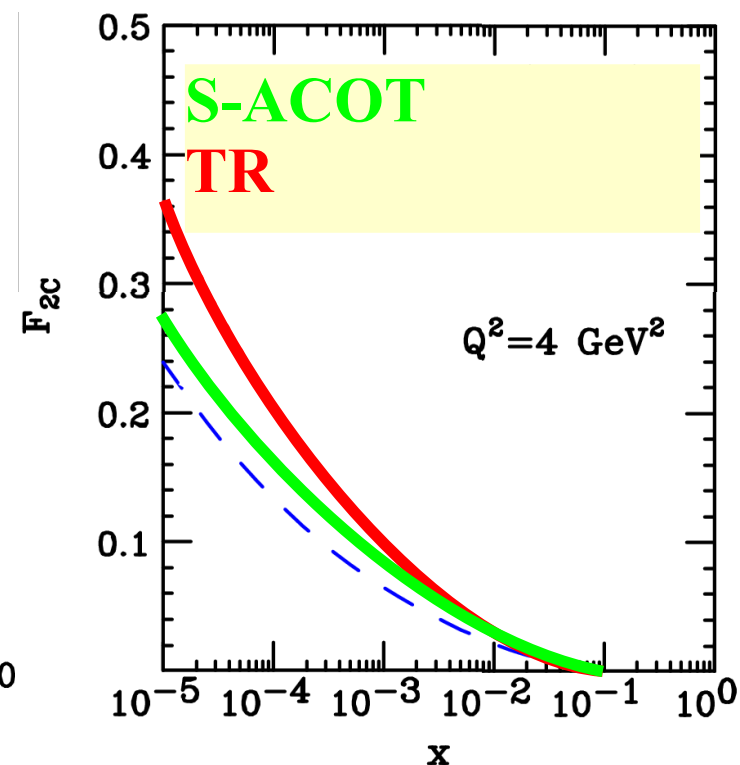
ACOT & S-ACOT  
essentially identical

... scheme  
differences are  
higher order



FONNL & S-ACOT

Numerically similar



MSTW09

We can quantify  
theoretical scheme  
differences

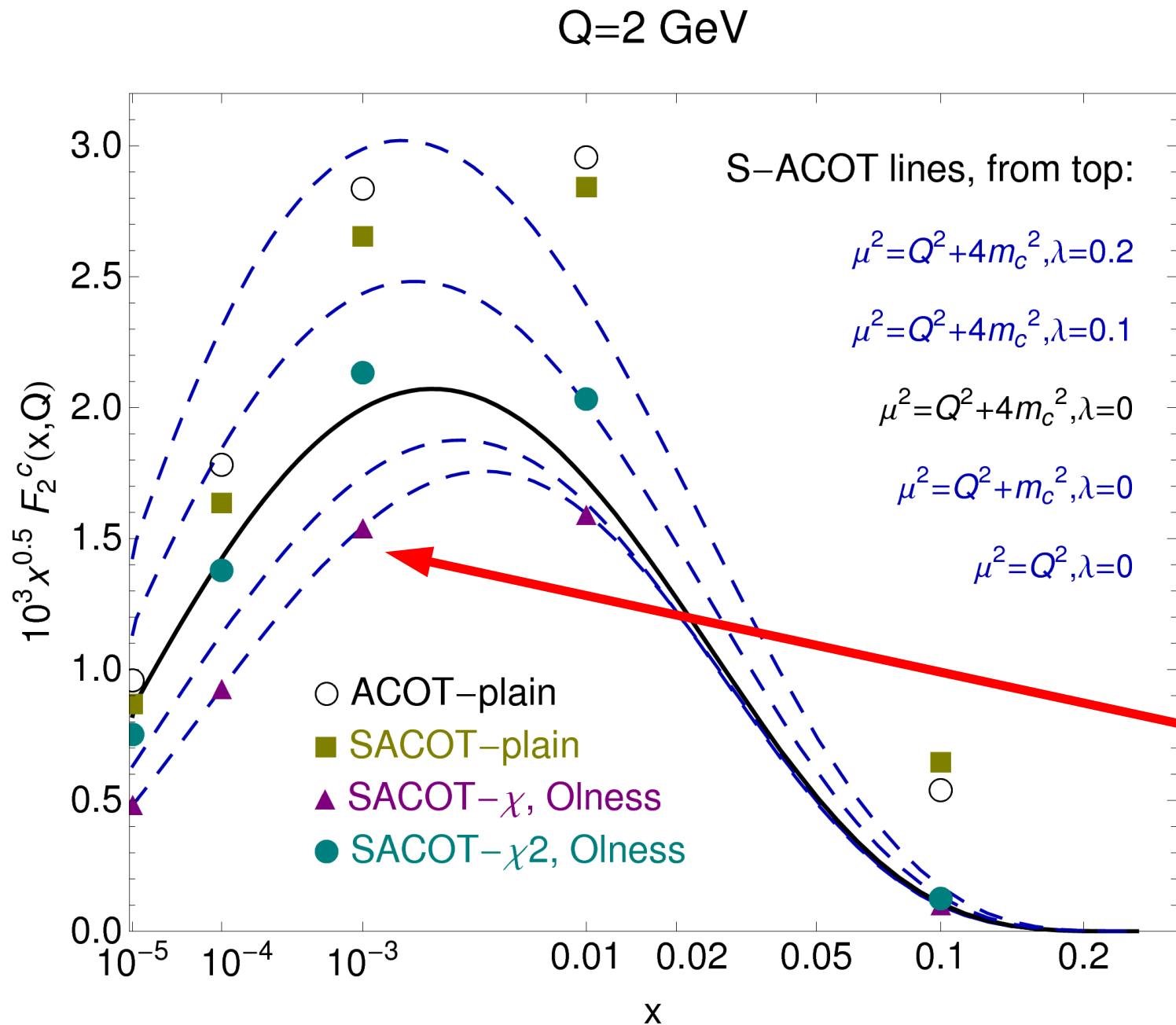
The SM and NLO Multileg Working Group: Summary report.

*J. Rojo, et al.,*

*e-Print: arXiv:1003.1241 [hep-ph]*



# Comparison of ACOT code with Nadolsky/Tung



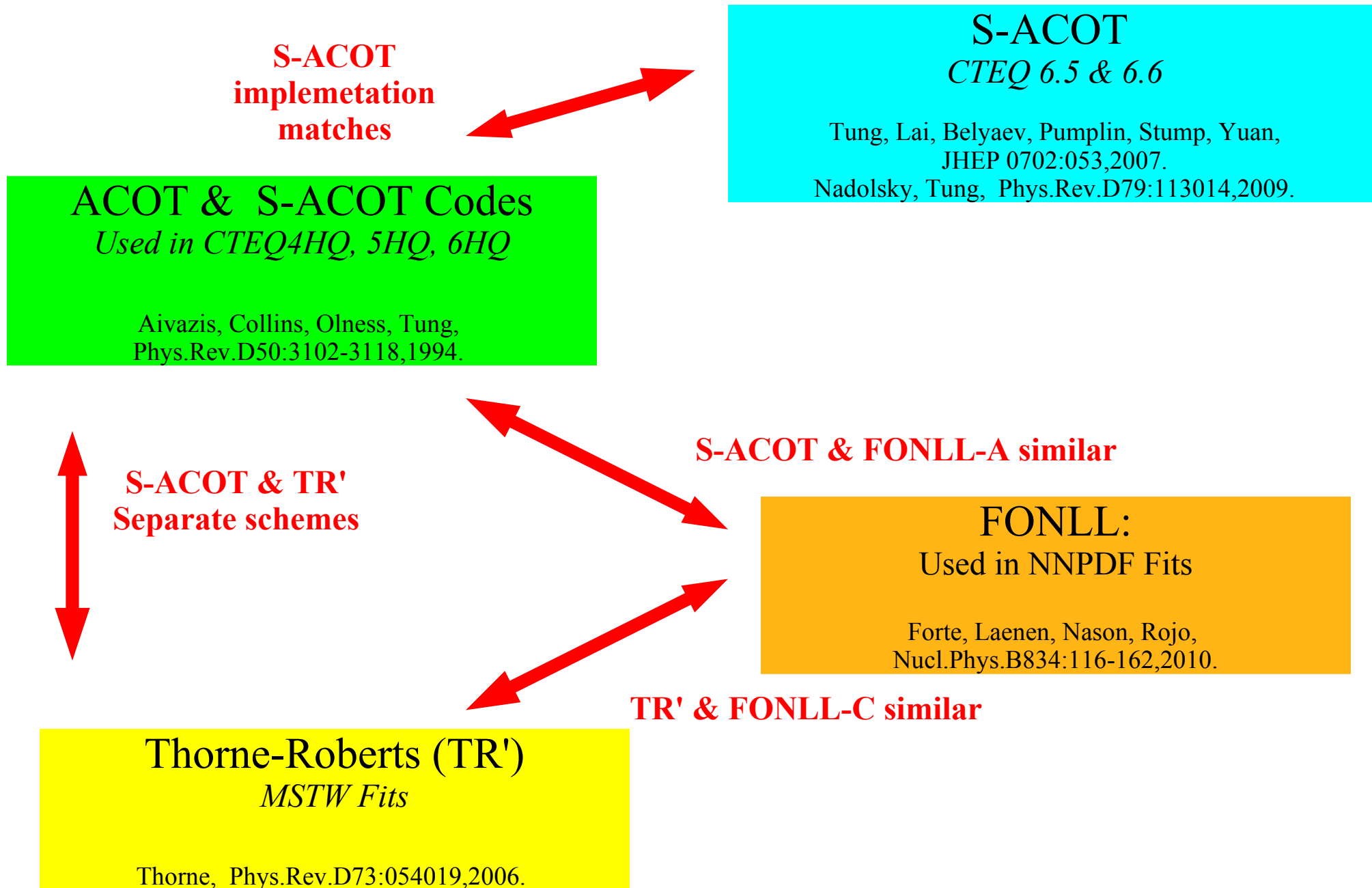
ACOT code  
&  
Nadolsky/Tung

**Results check**

Compare ▲ with  
bottom curve

$$x = \zeta (1 + \zeta^\lambda m^2/Q^2)^{-1}$$

# The Short Story:



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# Compare & Contrast ACOT & TR

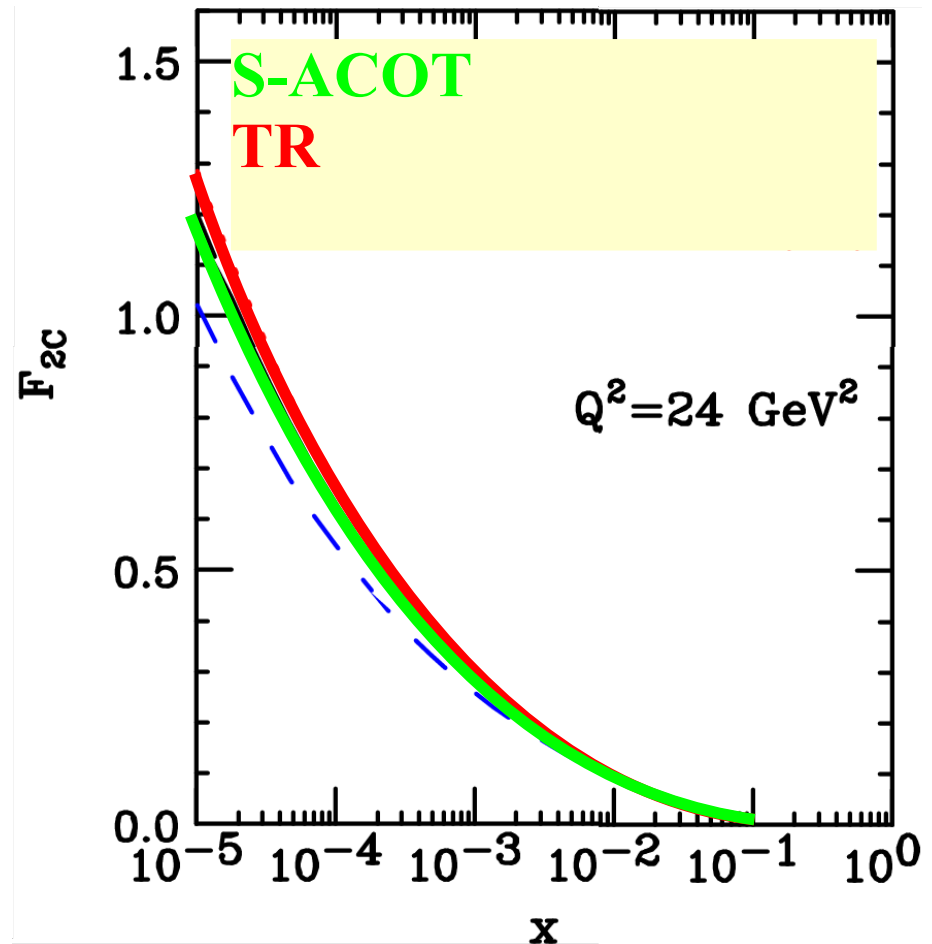
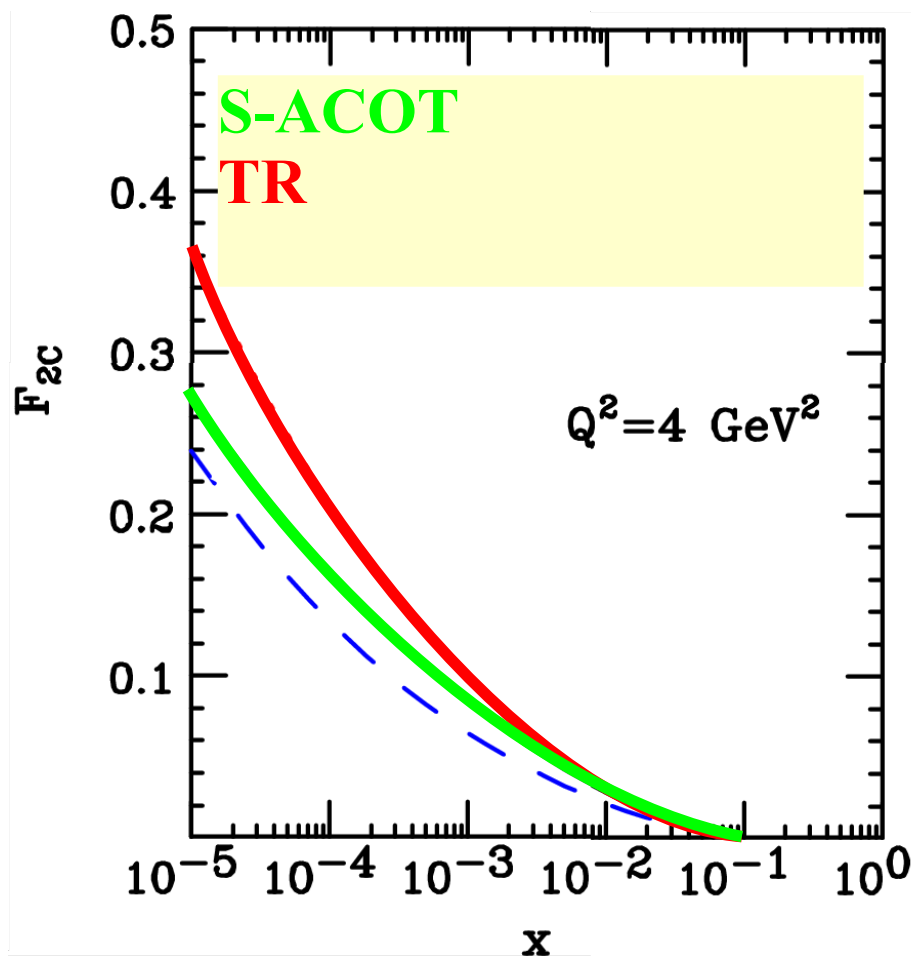
TR type schemes

ACOT type schemes

	$Q < m_H$	$Q > m_H$	constant term		$Q < m_H$	$Q > m_H$	constant term
LO					LO	$\emptyset$	+ $\emptyset$
NLO					NLO	+	+ $\emptyset$
NNLO					NNLO	+	+ $\emptyset$



## Comparison of ACOT & TR Schemes



Different schemes  $\Rightarrow$  Different PDFs  $\Rightarrow$  yet consistent  $\sigma$

Differences reduce at:

- 1) higher  $Q$ ,
- 2) higher order

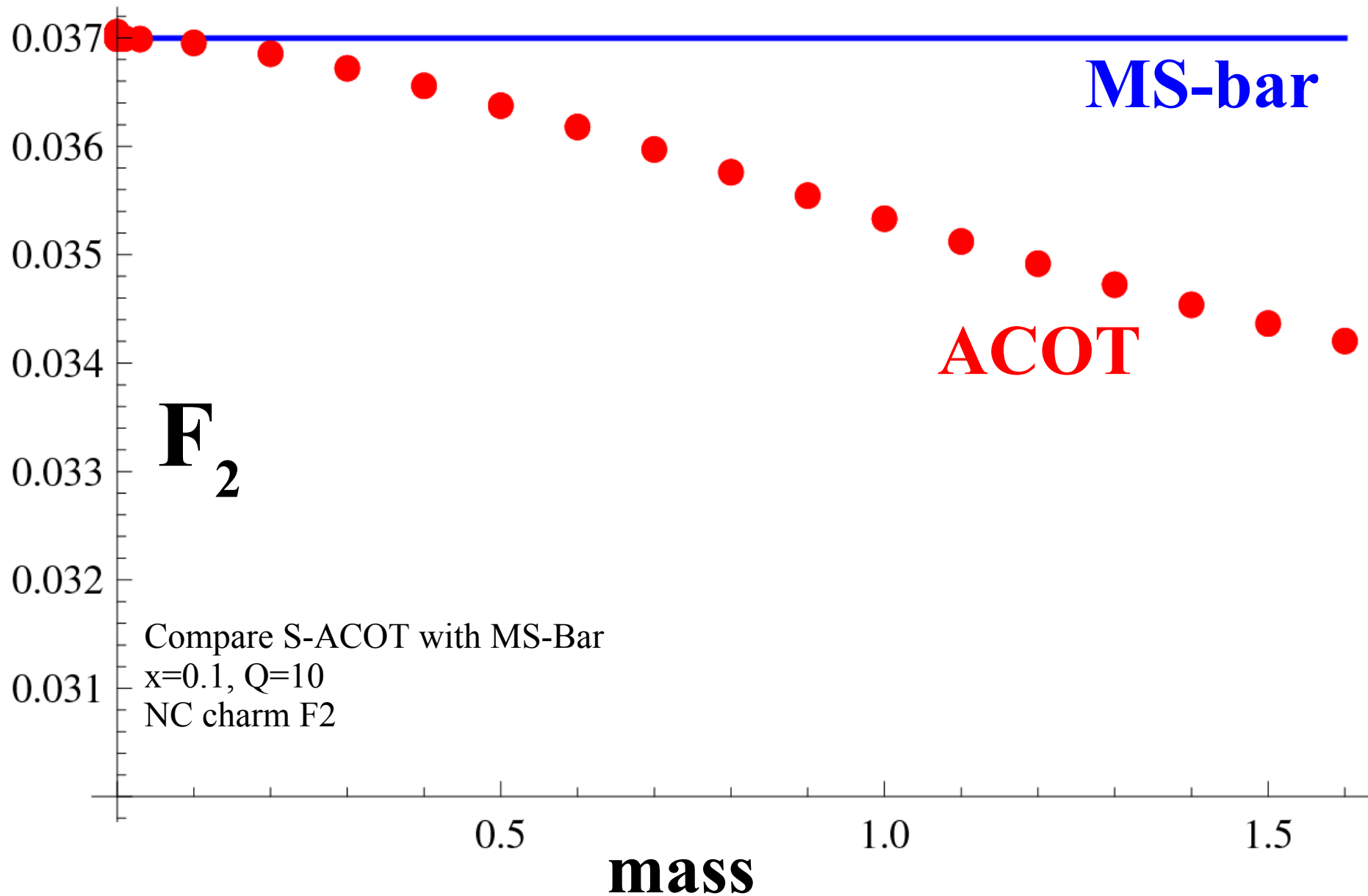
*If experiments are sensitive, time to compute to higher order*

ACOT:  $m \rightarrow 0$  limit yields MS-Bar  
*with no finite renormalization*

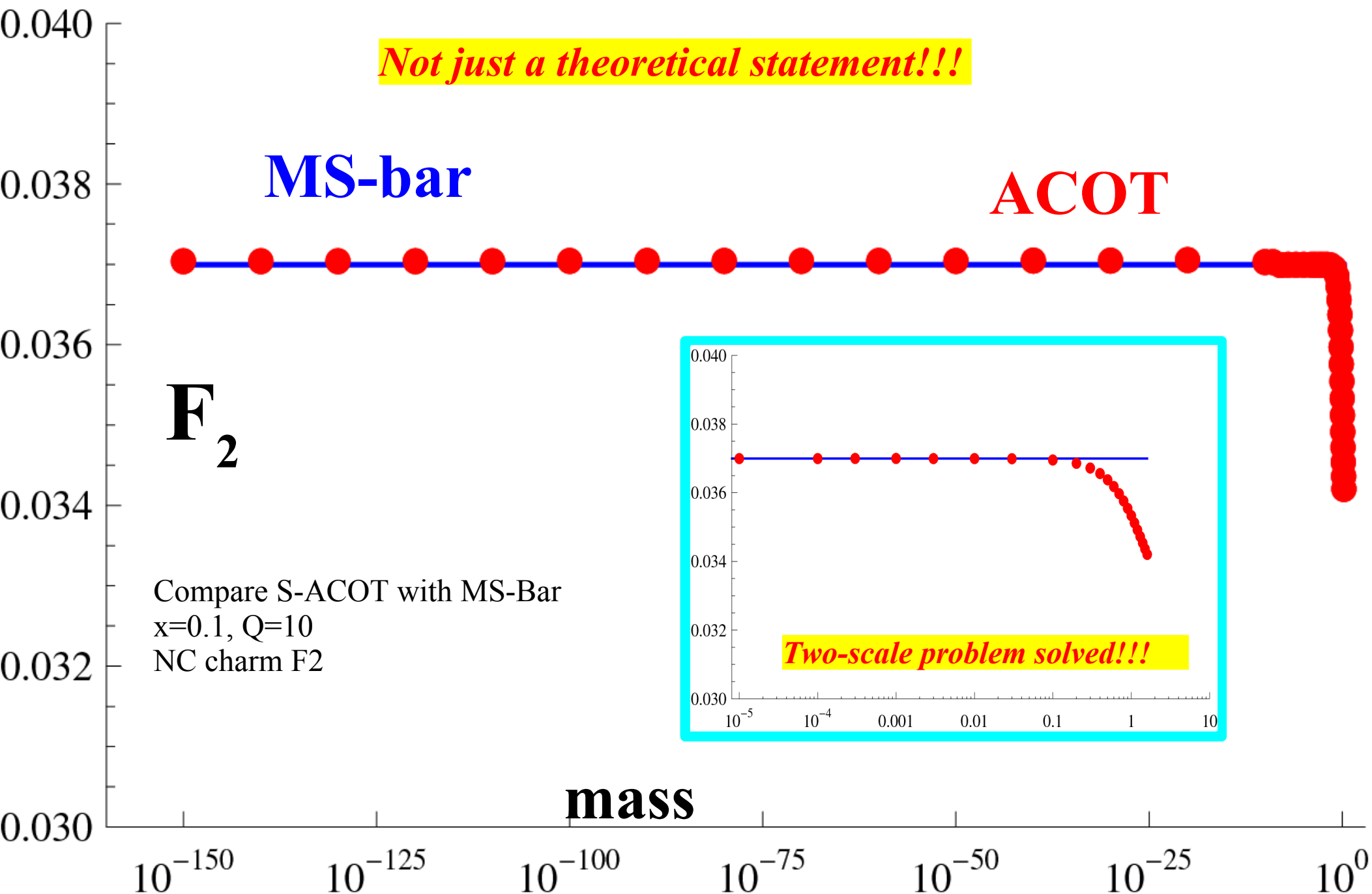
Based on the Collins-Wilczek-Zee (CWZ)  
Renormalization Scheme  
*... hence, extensible to all orders*

DGLAP kernels & PDF evolution are pure MS-Bar  
*Definition of Subtractions analogous to MS-Bar*

The minimal extension of MS-Bar scheme

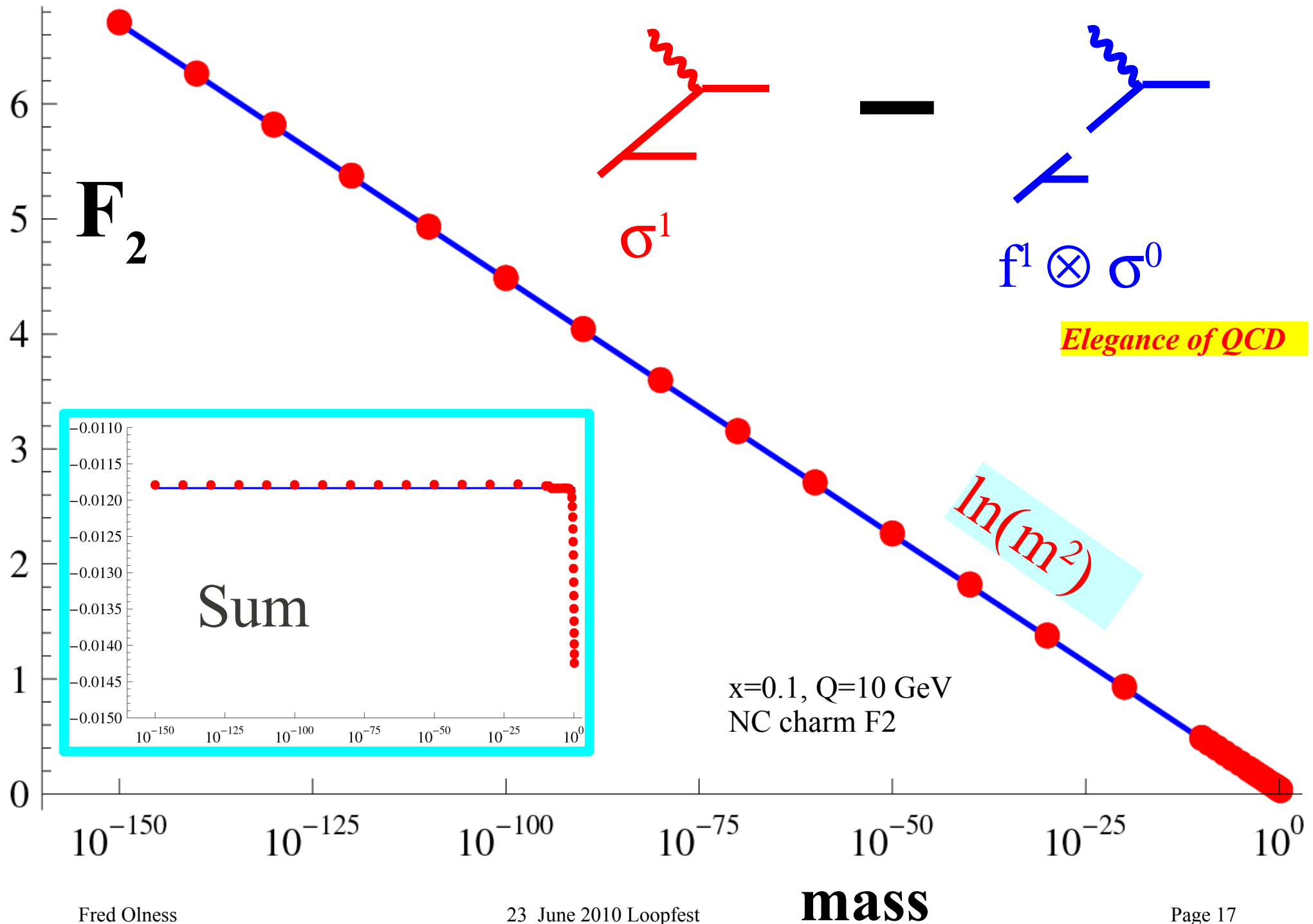


**ACOT  $m \rightarrow 0$  limit yields MS-Bar:** *No finite renormalization*





ACOT  $m \rightarrow 0$  limit yields MS-Bar: *No finite renormalization*



# Application of Factorization Formula at Leading Order (LO)

Basic Factorization Formula

$$\sigma = f \otimes \omega \otimes d + \mathcal{O}(\Lambda^2/Q^2)$$

Note: not  $m^2/Q^2$

**At Zeroth Order:**

$$\sigma^0 = f^0 \otimes \omega^0 \otimes d^0 + \mathcal{O}(\Lambda^2/Q^2)$$

Use:  $f^0 = \delta$  and  $d^0 = \delta$  for a parton target.

$f^0$

$f^1$

for parton target

Therefore:

$$\sigma^0 = f^0 \otimes \omega^0 \otimes d^0 = \delta \otimes \omega^0 \otimes \delta = \omega^0$$

$$\sigma^0 = \omega^0$$

*Z massive projection operators  
Collins (1998)*

**Warning:** *This trivial result leads to many misconceptions at higher orders*

# Application of Factorization Formula at Next to Leading Order (NLO)

## Basic Factorization Formula

$$\sigma = f \otimes \omega \otimes d + \mathcal{O}(\Lambda^2/Q^2)$$

## At NLO:

$$\sigma^1 = f^1 \otimes \omega^0 \otimes d^0 + f^0 \otimes \omega^1 \otimes d^0 + f^0 \otimes \omega^0 \otimes d^1$$

$$\sigma^1 = f^1 \otimes \sigma^0 + \omega^1 + \sigma^0 \otimes d^1$$



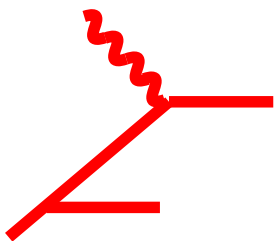
We used:  $f^0 = \delta$  and  $d^0 = \delta$  for a parton target.

$f^0$

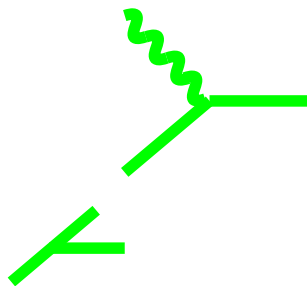
$f^1$

Therefore:

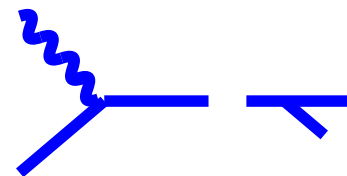
$$\omega^1 = \sigma^1 - f^1 \otimes \sigma^0 - \sigma^0 \otimes d^1$$



$\sigma^1$

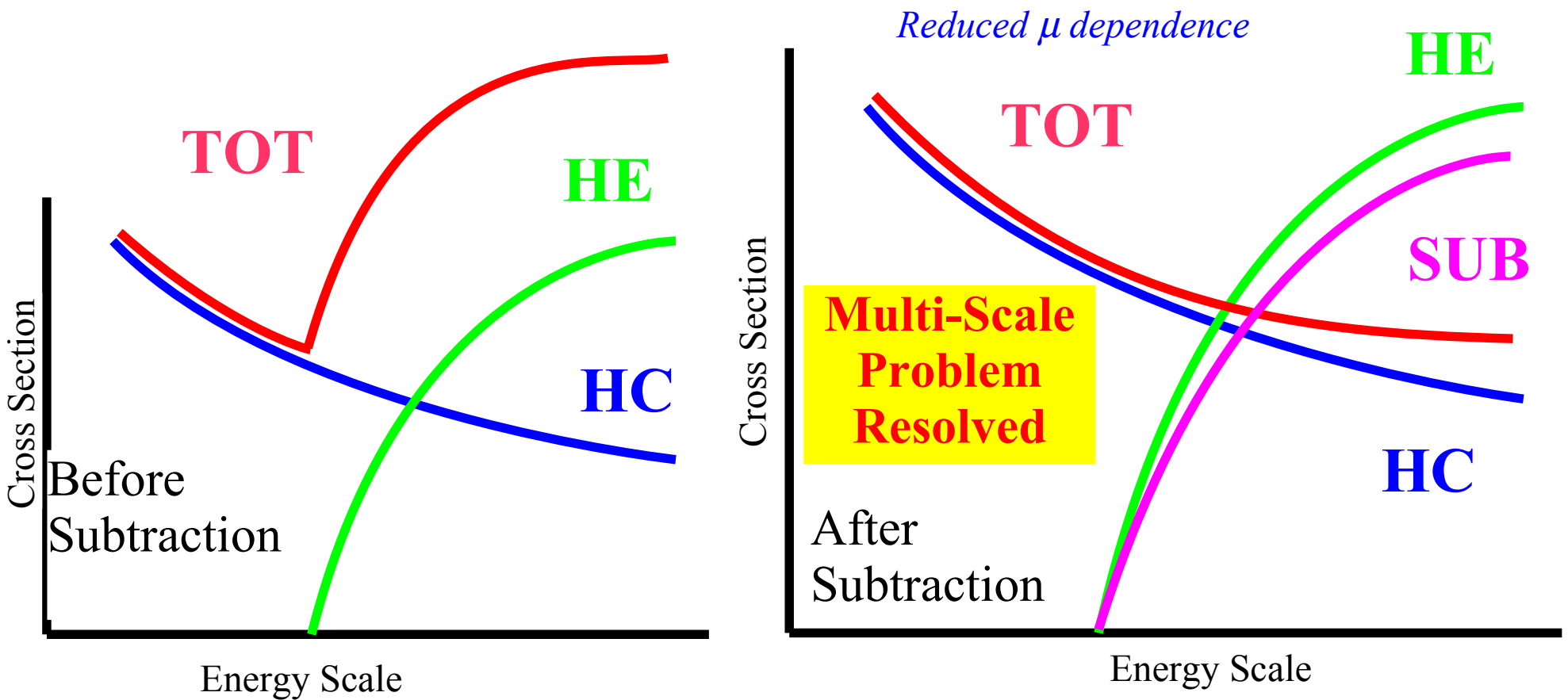


$f^1 \otimes \sigma^0$



$\sigma^0 \otimes d^1$

# Interaction of the separate contributions vs. energy scale



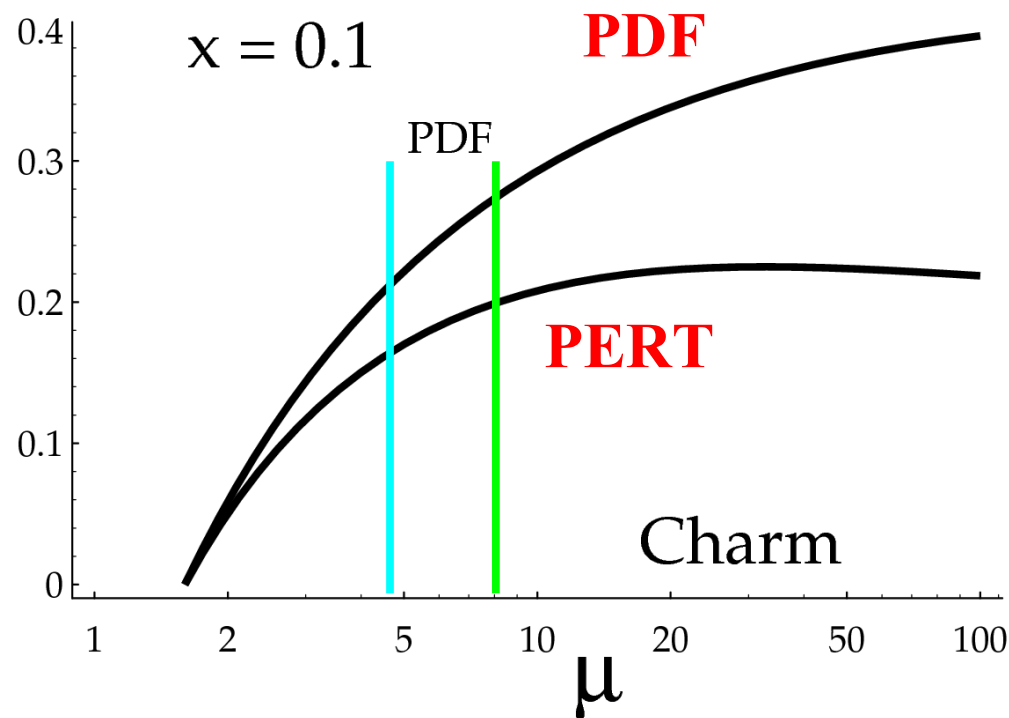
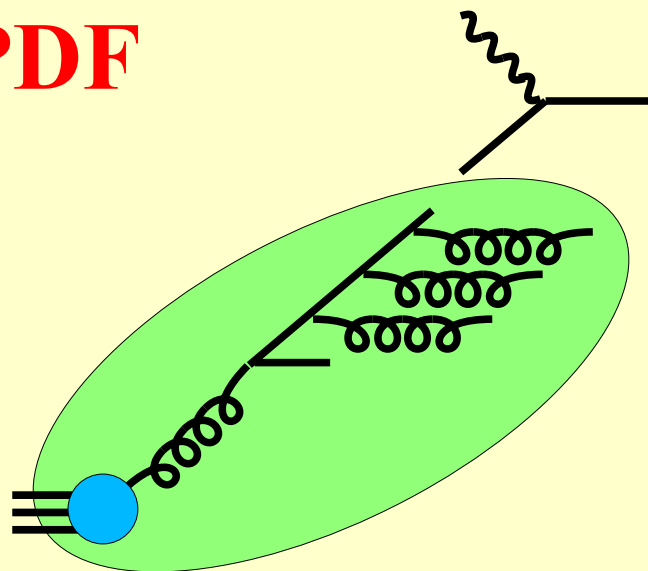
$$\text{TOT} = \text{Heavy Excitation} + \text{Heavy Excitation} - \text{Subtraction}$$

The diagram illustrates the components of the total cross section (TOT) as a function of energy scale. It shows the Heavy Excitation (HE) contribution (green), the Heavy Excitation (HC) contribution (blue), and the Subtraction (SUB) contribution (magenta). The equation indicates that the total cross section is the sum of the Heavy Excitation and Heavy Excitation contributions, minus the Subtraction contribution.

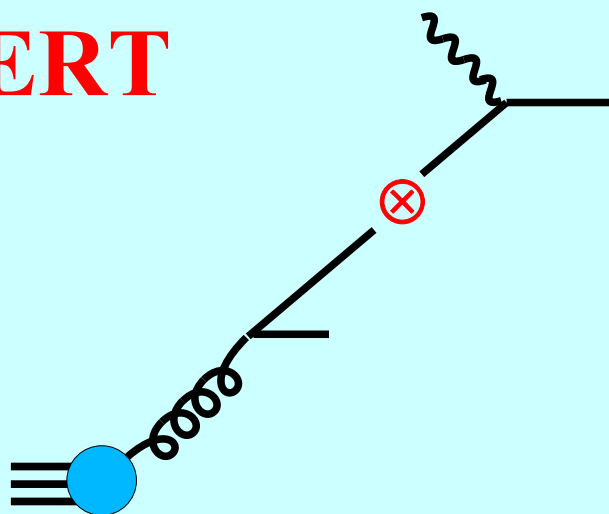


# Rule of Thumb: When do we need to consider heavy quark PDF???

**PDF**



**PERT**



**MORAL:** You don't have to choose which expansion point you use;

by using the Heavy Quark PDF,  
QCD will compensate

*In practice ...*

Using the heavy quark PDF's we can accommodate quark masses of any values: e.g.,  $10^{-150}$  to  $10^{+150}$

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# $\chi$ scaling

# $\chi$ -Scaling: Effect of Kinematic Mass Re-Scaling

**ACOT** (Aivazis, Collins, Olness, Tung) A general framework for including the heavy quark components.

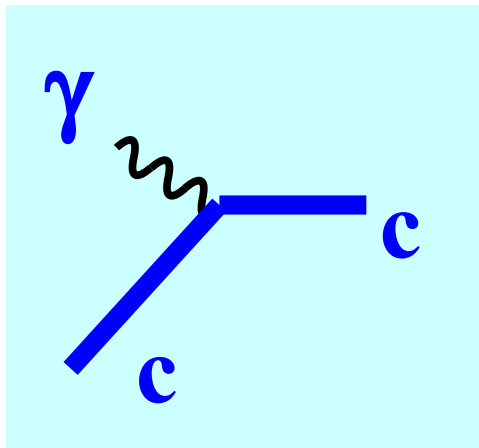
*Phys.Rev.D50:3102-3118,1994.*

**S-ACOT** (Simplified-ACOT) ACOT with the initial-state heavy quark masses set to zero.

*Phys.Rev.D62:096007,2000.*

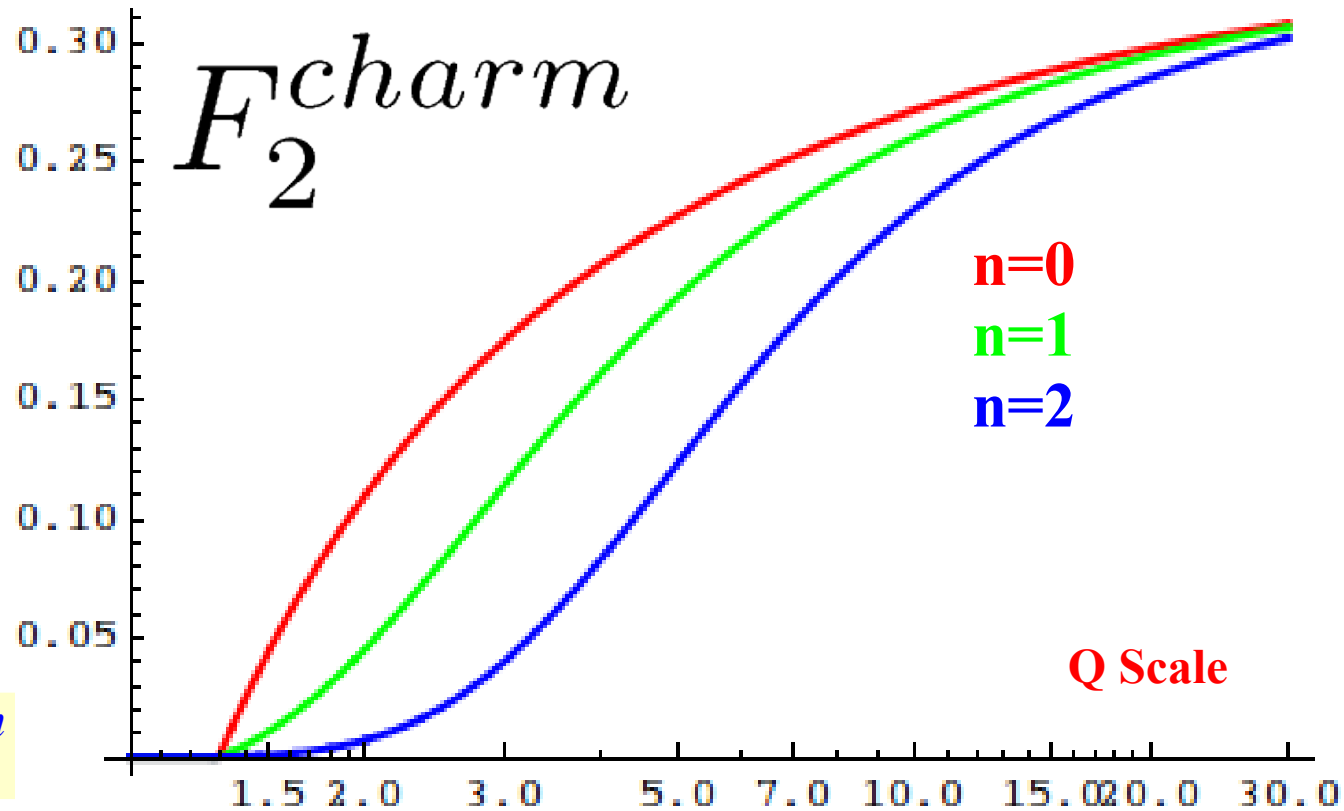
**ACOT- $\chi$  & S-ACOT- $\chi$ :** As above with a generalized slow-rescaling

*Phys.Rev.D62:096007,2000.*



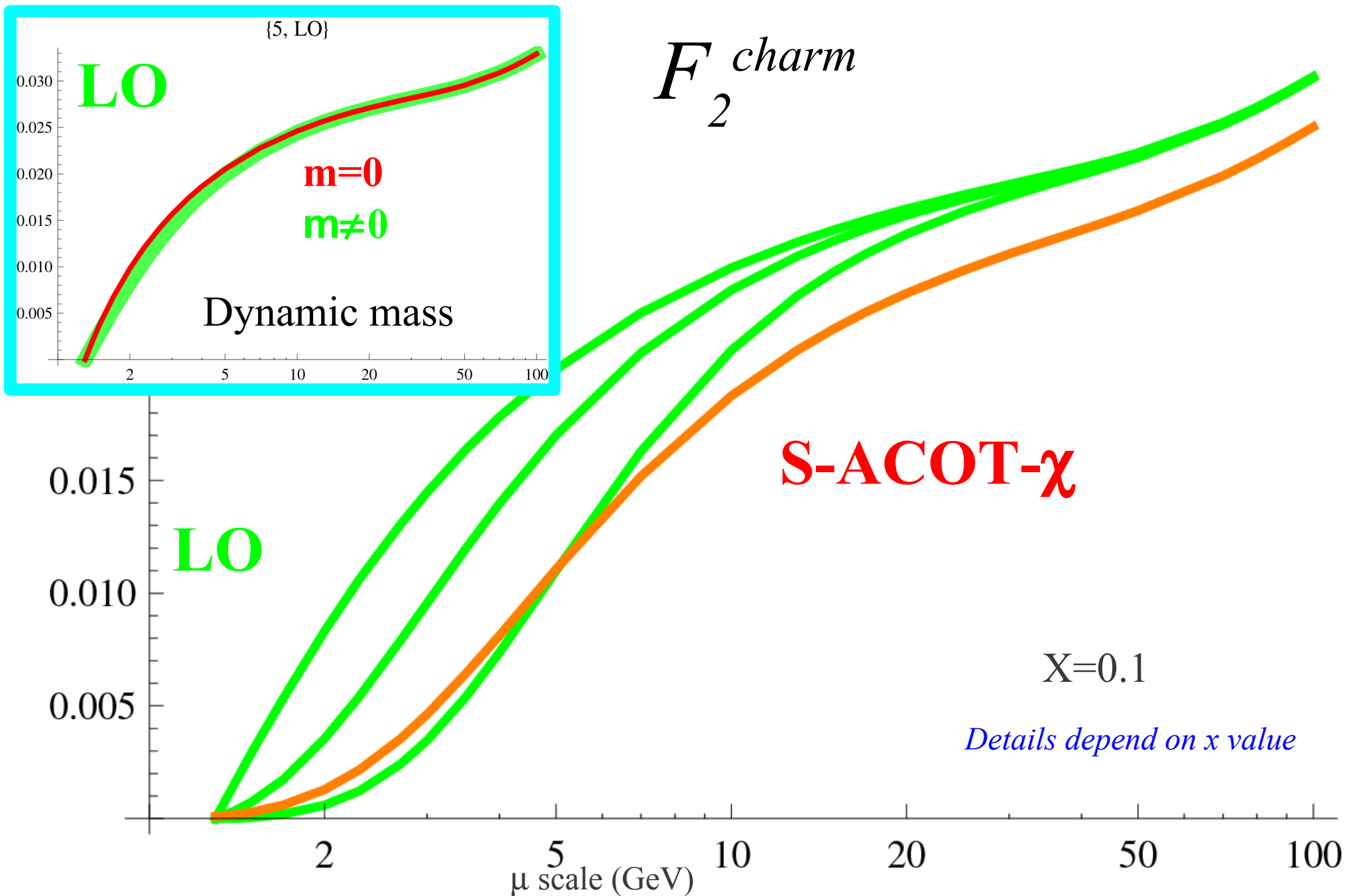
$$\chi = x \left[ 1 + \frac{(\mathbf{n}m_c)^2}{Q^2} \right]$$

*Technically, violates factorization  
e.g., D-meson beam*



***Kinematic Masses are more important than Dynamical Masses (in general)***

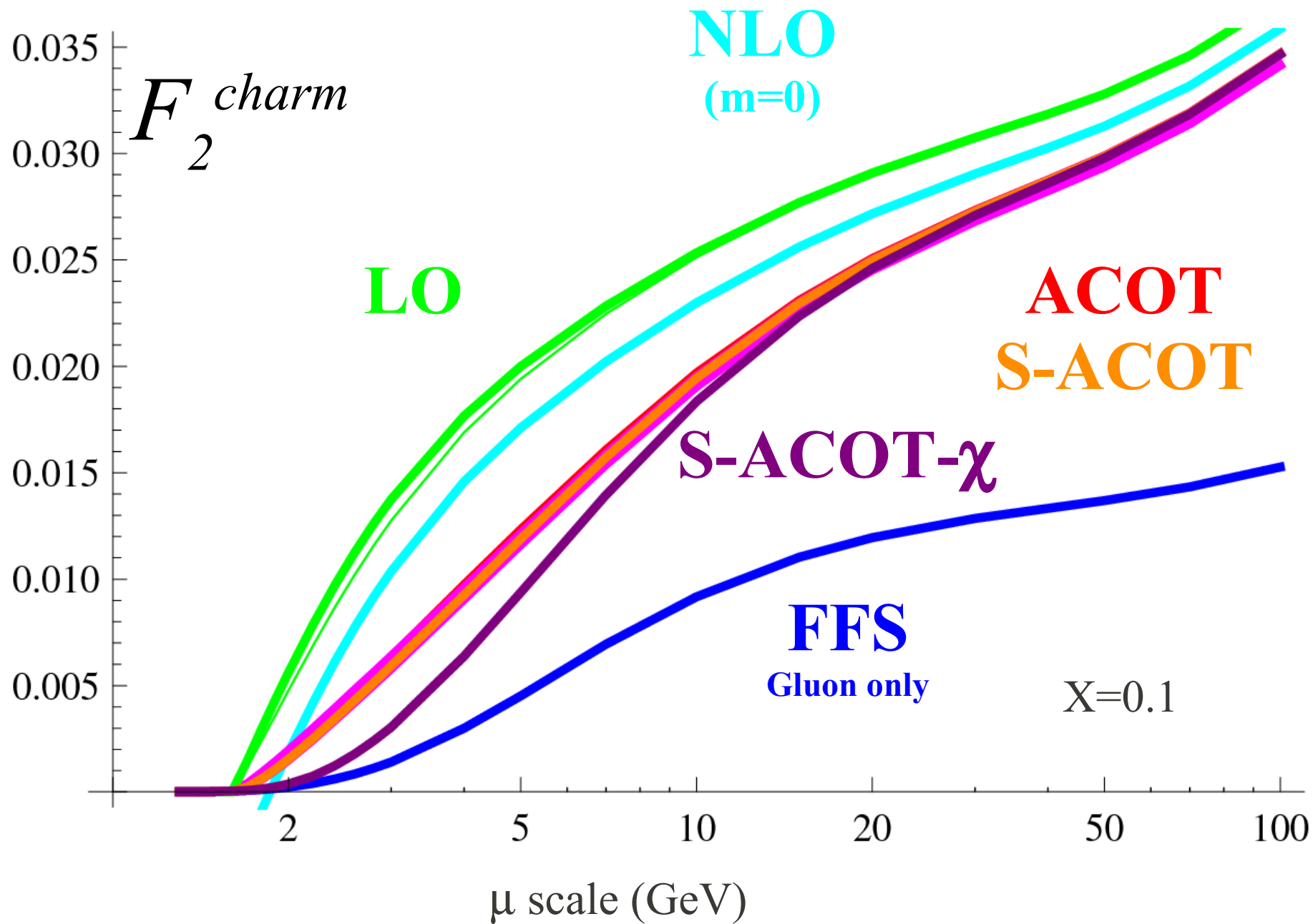
# $F_2$ Charm in the threshold region



***Kinematic Masses are more important than Dynamical Masses (in general)***



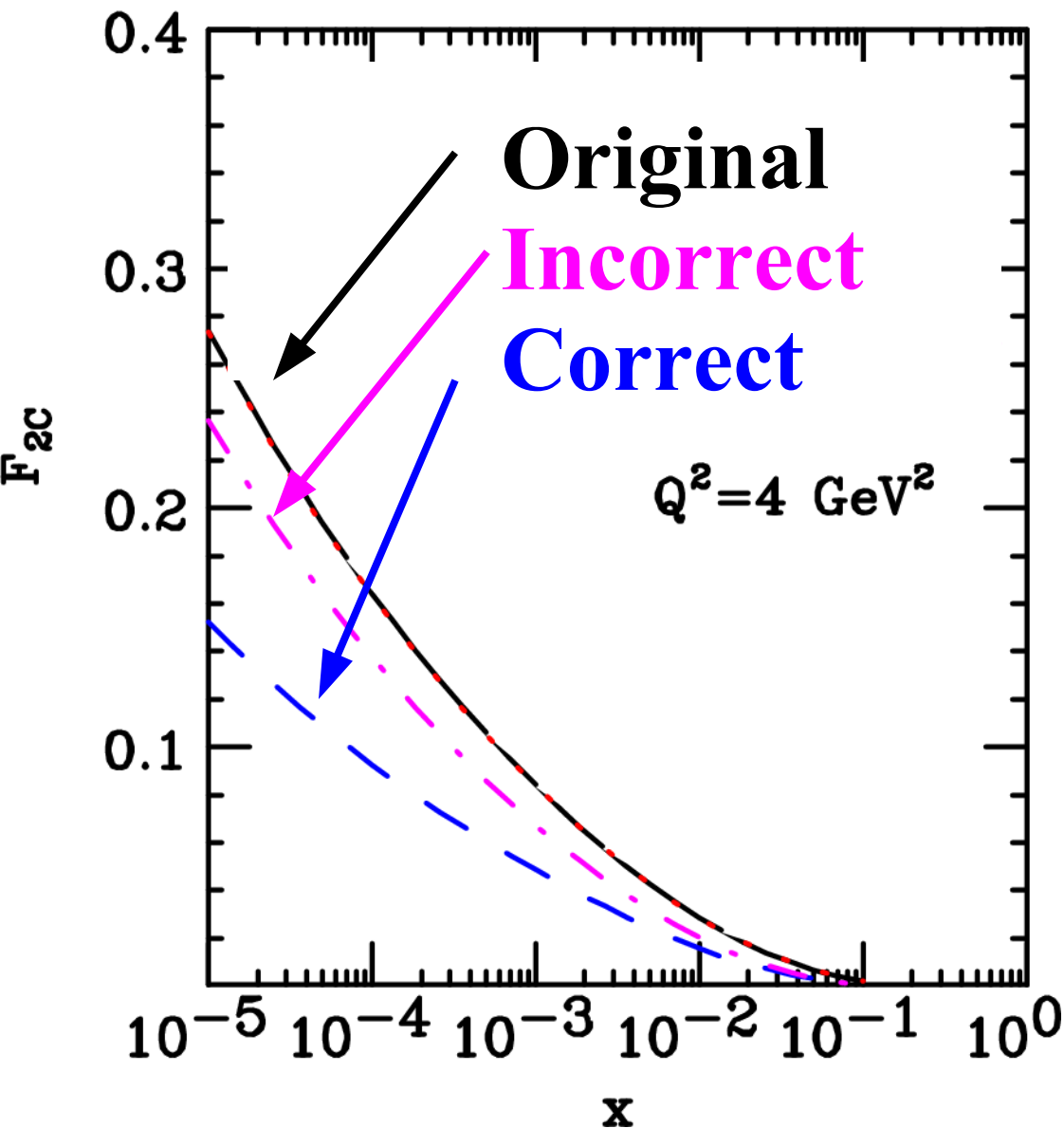
## $F_2$ Charm in the threshold region



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$F(\chi, Q)$       ???

Caution: Don't confuse  $xF_2(x, Q^2)$  and  $\chi F_2(\chi, Q^2)$



$\{x, Q\}$  is correct  
kinematic point

Recall  $\chi$  contains  
information of  
parton masses

*schematically*

$$F_2(x, Q) = x \int \dots f(x)$$

$$F_2(\chi, Q) = \chi \int \dots f(\chi)$$

$$F_2(x, Q) = x \int \dots f(x) \dots \delta(x - \chi)$$

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# NNLO

# Application of Factorization Formula at Next to Next to Leading Order NNLO

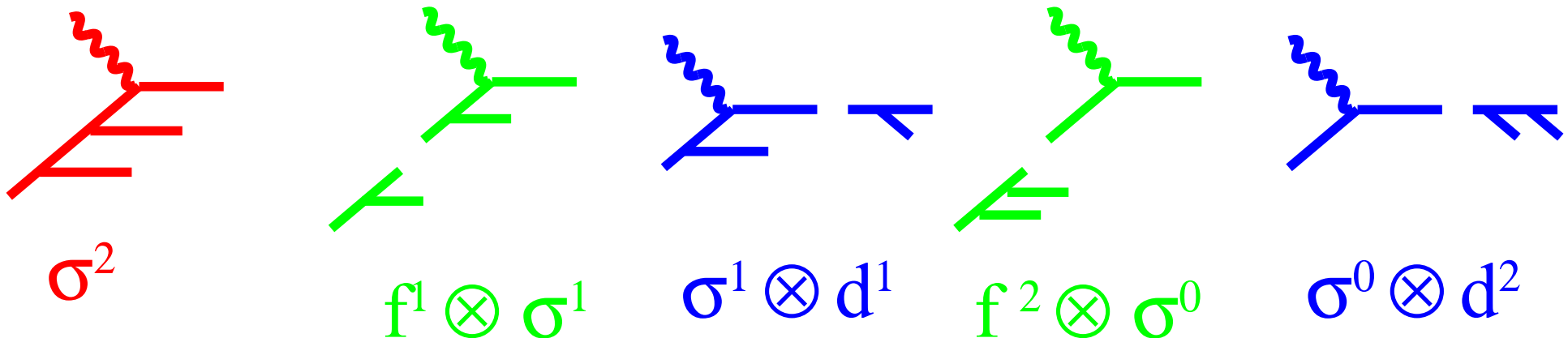
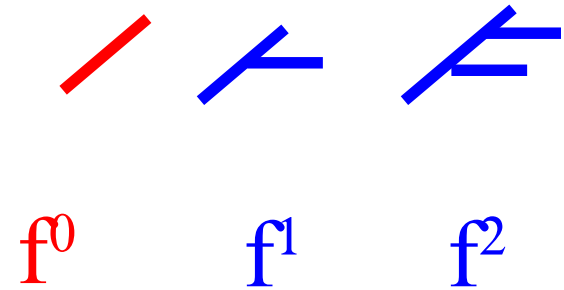
## Basic Factorization Formula

$$\sigma = f \otimes \omega \otimes d + \mathcal{O}(\Lambda^2/Q^2)$$

## At NNLO:

$$\sigma^2 = f^2 \otimes \omega^0 \otimes d^0 + f^0 \otimes \omega^2 \otimes d^0 + f^0 \otimes \omega^0 \otimes d^2 + f^1 \otimes \omega^1 \otimes d^0 + f^1 \otimes \omega^0 \otimes d^1 + f^0 \otimes \omega^1 \otimes d^1$$

We used:  $f^0 = \delta$  and  $d^0 = \delta$  for a parton target.



S-ACOT:

Works great at NLO  
Issues at NNLO

$\chi$  scaling:

Difficulties at NNLO  
Issues *even* at NLO,

*I showed you best case ( $x=0.1$ )*

In contrast, full ACOT:

- Extensible to all orders

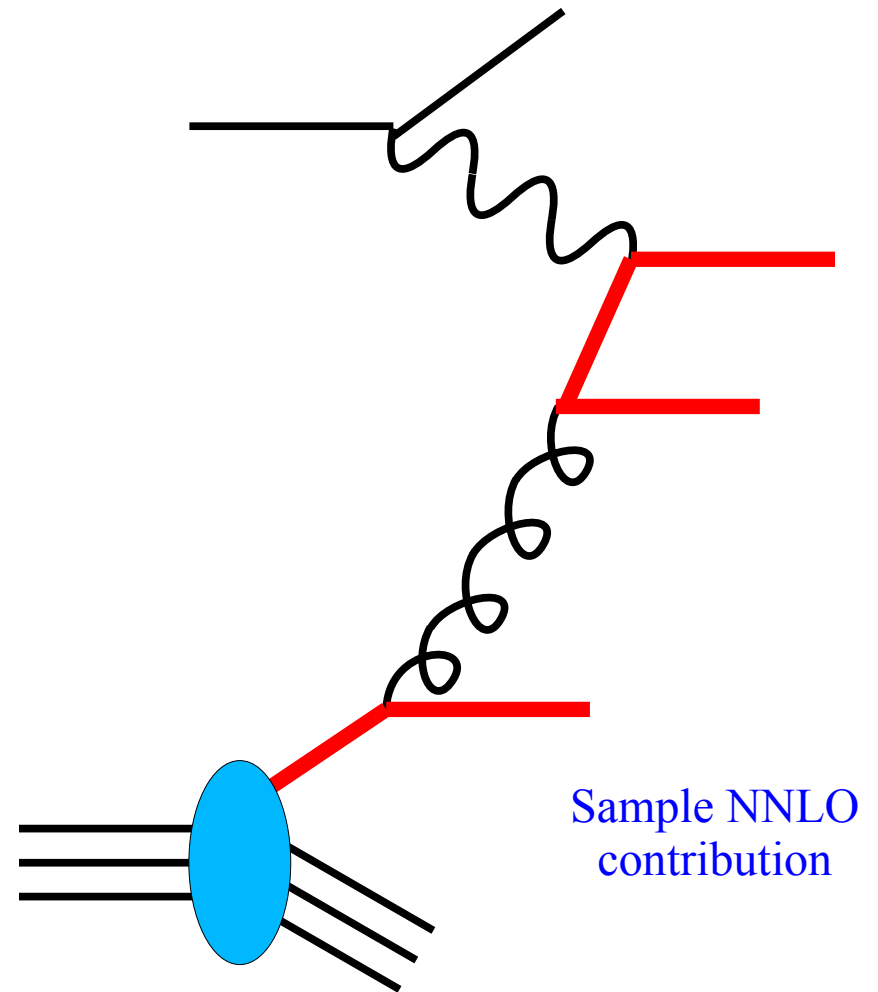
*Benefits from recent progress  
of higher order massive calculations*

- Includes masses in scaling variable ( $\chi$ )

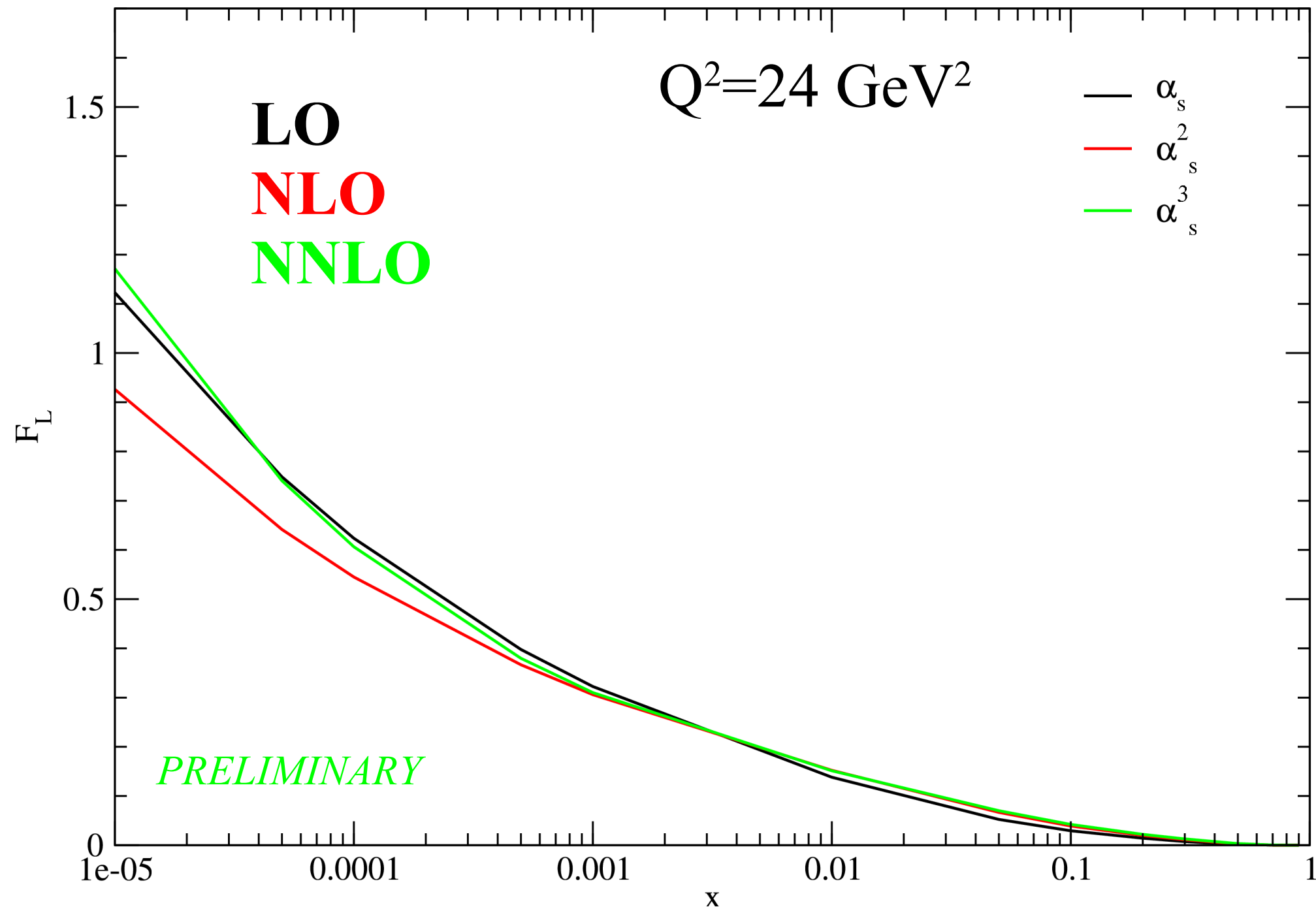
*Avoids  $m$  ambiguities at NNLO*

- Reduces to MS-Bar in  $m \rightarrow 0$  limit

*No finite renormalization terms*







## Heavy Quarks:

Essential to properly incorporate mass effects for required precision

Improved measurements of  $F^2$ ,  $F^{\text{cc}}$ ,  $F^{\text{bb}}$ , and  $F_L$ :

Improved precision for LHC where heavy flavors play a prominent role

## 2009 Les Houches Benchmark Comparisons:

Highlights recent progress

Important reference point

Shows theoretical scheme uncertainty

Comparisons enlightening

Theoretically, we can now compute full dynamic mass range  $[10^{-150}, 10^{+150}]$

ACOT natural massive extension of  $\overline{\text{MS}}$ -bar

Separate roles of dynamic and kinematic masses illustrated

Mass effects are essential:

Improvement, progresss, & understanding on theoretical side:

Thanks to: P. Nadolsky, K. Park, M. Guzzi I Schienbein, J.-Y. Yu, K. Kovarik, T.P. Stavreva

J. Owens, J. Morfin, C. Keppel, D. Soper ...

*& the Les Houches Working Group*